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Cosmological constraints from the masses and abundances of L_* galaxies

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ABSTRACT

We place limits on the mean density of the universe Ω_m and the effective slope of the linear power spectrum around a megaparsec scale $n_{\rm eff}$ by comparing the universal mass function to the observed luminosity function. Numerical simulations suggest that the dark matter halo mass function at small scales depends only on $\Omega_m(n_{\rm eff}+3)$ independent of the overall power spectrum normalization. Matching the halo abundance to the observed luminosity function requires knowing the relation between the virial mass and luminosity (separately for early and late type galaxies) and the fraction of galaxies that reside in larger halos such as groups and clusters, all of which can be extracted from the galaxy-galaxy lensing. We apply the recently derived values from SDSS and find $\Omega_m(n_{\rm eff}+3)=(0.15\pm0.05)/(1-f_{\rm dh})$, where $f_{\rm dh}$ accounts for the possibility that some fraction of halos may be dark or without bright central galaxy. A model with $\Omega_m=0.25$ and primordial $n=0.8$ or with $\Omega_m=0.2$ and $n=1$ agrees well with these constraints even in the absence of dark halos, although with the current data somewhat higher values for Ω_m and n are also acceptable.

1 INTRODUCTION

Halo mass function has been long recognized as a powerful probe of cosmology since the seminal work by Press & Schechter (1974). Most of the applications so far have focused on clusters, which are easy to detect in X-rays and for which the observed X-ray temperatures correlate well with the cluster mass. While there is currently some uncertainty in the normalization of this relation, upcoming X-ray and weak lensing observations should provide empirical means to calibrate it. Since the clusters are the most massive halos formed in the universe they lie on the exponential tail of the mass function, whose amplitude depends mainly on the overall normalization of the power spectrum and the density parameter Ω_m. On the other hand, for the halo mass below the nonlinear mass the mass function depends only on the density of the universe and on the effective slope of the linear power spectrum at that scale and is independent of the power spectrum normalization (Press & Schechter 1974, Sheth & Tormen 1999, Jenkins et al. 2001).

To constrain these cosmological parameters with mass function one must be able to determine masses and abundances of halos below the cluster scale. Groups with masses between $10^{13}-10^{14}M_\odot$ are difficult to observe directly, since they contain only a handful of galaxies, and their abundance is quite uncertain in this range. Below $10^{11}M_\odot$ many of the halos may not host a bright galaxy because of effects that prevent either gas cooling or star formation, such as UV background radiation, feedback or insufficient surface density for star formation. Moreover, a significant fraction of galaxies corresponding to these halo masses may be satellites inside a larger halo and one must correct for that, since the halo mass function only counts isolated halos. Estimating this fraction is difficult for small halos.

In the mass range $10^{11}-10^{12}M_\odot$, typical for L_* galaxies, the luminosity function is well determined and theoretical models suggest that each of these halos should host a bright galaxy at the center (e.g. Kauffmann et al. 1999, Benson et al. 2000). The main challenge in this range is to determine the relation between a galaxy luminosity and a halo mass and the fraction of these galaxies that belong to larger halos such as groups and clusters. Even though we have good dynamical probes of mass within the optical region of a galaxy, such as Tully-Fisher relation for late types and Faber-Jackson relation or strong lensing for early types, the relation between optical masses and virial masses is more uncertain, since the virial mass depends on the adopted density profile and a typical virial radius is a factor of 10 larger than optical radius. Virial masses thus cannot be observationally constrained from the optical or HI studies alone, which was used in previous work on the mass function determination on galactic scales (Gonzalez et al. 2000, Kochanek 2001).

An alternative approach adopted here is to use virial masses derived directly from the galaxy-galaxy (g-g) lensing. In this method one uses tangential distortions of background galaxies by the foreground galaxy to place limits on the mass distributions around galaxies. A recent SDSS analysis presents galaxy-galaxy lensing results using a much larger sample than previously available, allowing a detailed study of the relation between mass and light for several luminosity

bands and morphological types (McKay et al. 2001). The latter is particularly important since the relation between mass and luminosity differs significantly between early and late type galaxies and one must extract the relations separately before combining them together. The data are most sensitive to 100-200h^{-1}kpc transverse separations, which is a typical virial radius of a $10^{11}-10^{12}h^{-1}M_\odot$ halo. Second complication is the fraction of galaxies that are not in isolated halos. This can also be extracted from g-g lensing: if there is a significant lensing signal around the galaxies at separations above $200h^{-1}$kpc then it signals a presence of groups and clusters around them (Guzik & Seljak 2002). This allows one to determine the fraction of galaxies of a given luminosity in these larger halos. Together thus g-g lensing provides all the necessary information needed for a quantitative study of halo mass function on galactic scales.

It is worth comparing this approach to the traditional mass to light ratio (M/L) method to obtain the density parameter Ω_m (e.g. Bahcall et al. 1995, Carlberg et al. 1997). This often assumes that M/L extracted from some type of objects, such as groups or clusters, can be applied to the global M/L. Since we can measure the total luminosity density in the universe we can obtain the total matter density by multiplying the two. However, M/L depends on both the halo mass and scale on which one measures it. While light is concentrated to the inner parts of the clusters mass continues to increase, so M/L for any given object increases with radius. Even adopting the virial masses, defined so that only baryons within that radius can condense to make stars, there is no reason why $M_{\rm vir}/L$ should not depend on L. Theoretical models predict $M_{\rm vir}/L$ to be at the minimum at galactic masses, where cooling and star formation are most efficient (e.g. Kauffmann et al. 1999, Benson et al. 2000). It is possible that there are many small halos that have no light at all (e.g. Jimenez et al. 1997), which is why we cannot see them, but which contribute to the mass of the universe. Moreover, there may be mass in the universe that is not associated with any collapsed structures at all. For current generation of simulations only about 40% of the total mass has been resolved into halos above $10^{11}h^{-1}M_\odot$ (Jenkins et al. 2001). It is not obvious that as the resolution of simulations increases this fraction converges to unity, which is what is commonly assumed in the forms of mass function (Press & Schechter 1974, Sheth & Tormen 1999), since some fraction of the mass could remain in a diffuse form. All this makes any extrapolation of $M_{\rm vir}/L$ to the total luminosity density as a method to deduce the density of the universe highly uncertain. One can attempt to correct for this using simulations (e.g. Bahcall et al. 2000), but this relies on the ability of these to reproduce the light distribution in the universe across a large dynamic range of masses and scales. This is one venue to pursue in the future as simulations and modelling of physical processes improve.

The alternative way is to measure mass and light over a much larger volume, so that it becomes representative for the whole universe. Currently there are no reliable methods that can measure the mean mass directly on such large scales. The closest method to achieve this goal is gravitational lensing, which measures ellipticity distortions of background galaxies. This method however cannot measure the mean component of the matter, which does not produce shear. So instead one must look at fluctuations around the mean by comparing the relation between galaxy light and convergence (Wilson et al. 2001). However, if galaxies are a biased tracer of dark matter then only a combination Ω_m/b can be determined and if $b>1$ this will underestimate Ω_m. Since this is a luminosity weighted statistic it will give a larger weight to brighter galaxies, which are known to be biased relative to fainter ones (e.g. Norberg et al. 2001; Zehavi et al. 2001).

Since it is difficult to detect a signal on large scales with gravitational lensing, where linear biasing applies, one must compare the fluctuations in light and mass at smaller scales. At any given scale halos of a certain mass dominate the fluctuations (see e.g. Seljak 2000 for a discussion of this in the context of halo models). For example, on a megaparsec scale the dominant fluctuations come from groups and clusters, while on 50kpc scale the galactic halos dominate the fluctuations. This means that even if the galaxies are unbiased the relation between light and mass will be appropriate only for the halos that dominate the fluctuations at that smoothing scale. As discussed above this mass to light relation may not be the universal one, since most of the mass may be in smaller objects or in diffuse structures, which have too weak signal to be detected through the weak lensing fluctuations. There is no preferred scale at which one should evaluate M/L to multiply it with the total galaxy light to derive the mean density of the universe. If M/L increases with halo mass above L_*, as suggested by observations (Girardi et al. 2001, Guzik & Seljak 2002), then the convergence-light correlation function will be more extended than that of light itself and there is some observational evidence for this effect (Wilson et al. 2001). It is not clear however whether this leads to an underestimate or overestimate of Ω_m, since the trend of M/L to increase with mass is likely to be reversed below L_* and a lot mass associated with either diffuse structures or small halos may not be associated with any light at all.

Above arguments suggest that global mass to light ratio method of determining the density of the universe cannot be derived without making additional assumptions. In the approach presented here we instead limit the analysis only to the galaxies around L_* which are well studied. We combine the virial mass to luminosity relations extracted from the SDSS data with the SDSS luminosity function of L_* galaxies and compare that to the universal mass function to place limits on cosmological models. The analysis is done entirely within the SDSS data set, which reduces the uncertainties related to the photometric calibrations and color transformations, which usually plague luminosity function comparisons.

2 RELATION BETWEEN GALAXY AND HALO ABUNDANCES

The halo mass function describes the number density of halos as a function of mass. It can be written as

$$\frac{dn}{d\ln M}=\frac{\bar\rho}{M}f(\sigma)\frac{d\ln\sigma^{-1}}{d\ln M}, \tag{1}$$

where $\bar\rho$ is the mean matter density of the universe, M is the virial mass of the halo and $n(M)$ is the spatial number density of halos of a given mass M. We introduced a function $f(\sigma)$, which has a universal form independent of the

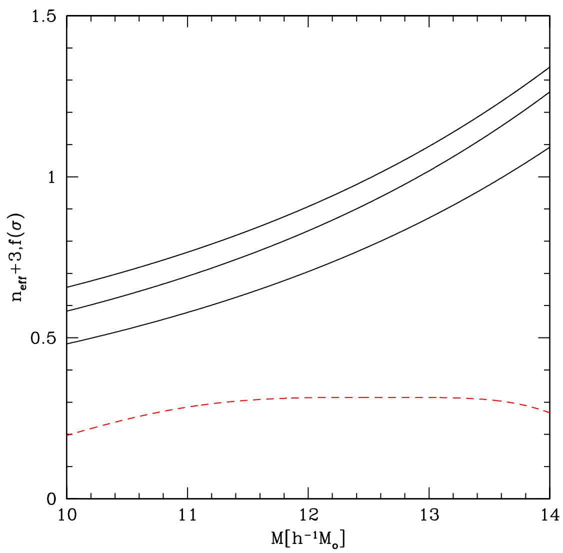

Figure 1. $n_{\rm eff}+3$ for models with $\Omega_m = 0.3$, $\Omega_b = 0.04$, $h = 0.7$, $n = 1$ (solid, top), $n = 0.9$ (solid, middle) and $\Omega_m = 0.25$, $\Omega_b = 0.04$, $h = 0.65$, $n = 0.8$ (solid, bottom). All the transfer functions were computed using CMBFAST. Also shown is $f[\sigma(M)]$ (dashed), which is essentially 0.3 over this range of masses.

power spectrum, matter density, normalization or redshift if written as a function of rms variance of linear density field,

$$\sigma^2(M) = 4\pi \int P(k) W_R(k) k^2 dk. \tag{2}$$

Here $W_R(k)$ is the Fourier transform of the spherical top hat window with radius R, chosen such that it encloses the mass $M = 4\pi R^3 \bar{\rho}/3$ and $P(k)$ is the linear power spectrum.

The relation between mass and rms variance depends on the linear power spectrum. For a smooth power spectrum one can locally approximate it as $P(k) \propto k^{n_{\rm eff}}$ and the relation is

$$\frac{d \ln \sigma^{-1}}{d \ln M} = \frac{n_{\rm eff}+3}{6}. \tag{3}$$

For CDM models on galactic scales the effective slope $n_{\rm eff}$ ranges between -1.5 and -2.5 (figure 1).

The universality of the mass function has been recently investigated by a number of authors (Sheth & Tormen 1999, Jenkins et al. 2001, White 2001). It has been shown that the mass function is indeed universal for a broad range of cosmological models. Jenkins et al. (2001) propose the following form,

$$f(\sigma) = 0.315 \exp[-|\ln \sigma^{-1} + 0.61|^{3.8}], \tag{4}$$

which they argue is universal if mass is expressed in terms of the virial radius where overdensity is 200 in units of mean density. It is remarkable that the mass function in this form is almost constant for all halos with $\ln \sigma > 0$. Here we are interested in halos on galactic scales where $\ln \sigma \sim 1$, so this limit applies and we can take $f \sim 0.3$, a universal value which varies only weakly with halo mass, as shown in figure 1 (dashed line).

To relate the theoretical halo abundance with the observed galaxy abundance we must assume a relation between the galaxy luminosity and its halo mass. Here we use a direct probe of the halo virial mass obtained from the galaxy-galaxy lensing of SDSS (McKay et al. 2001). Detailed modelling of CDM profiles shows that these observations are best fitted with L_* galaxies (which dominate the luminosity distribution of that sample) having a virial mass around $10^{12}h^{-1}M_\odot$ (Guzik & Seljak 2002). The virial mass strongly depends on the morphology or color: for a given luminosity early type galaxies can be significantly more massive than late type galaxies. The differences are reduced in red bands (r', i' and z'), where we focus our analysis as well, but they can still vary by a factor of 2-3. This is to a large extent a consequence of stellar age, which modifies the stellar mass to light ratio by a similar factor, with early type galaxies having a higher stellar mass to light ratio than late type galaxies. Current data are consistent with stellar mass to virial mass being independent of stellar age (or morphology). Nevertheless, in the absence of stellar mass information for individual galaxies, one must extract all the relations separately for the two types before adding them together. For example, for an $L_* = 2.1 \times 10^{10} h^{-2} L_\odot$ in i' (Blanton et al. 2001) one finds virial $M_{200\Omega_m} = 4.3 \times 10^{11} h^{-1} M_\odot$ for late type galaxies and $M_{200\Omega_m} = 1.2 \times 10^{12} h^{-1} M_\odot$ for early type galaxies, where we have corrected for a 25% increase from M_{200} to $M_{200\Omega_m}$ assuming $\Omega_m = 0.3$ fiducial model.

An alternative and more model dependent estimate of virial masses comes from the Tully-Fisher (TF) relation for late type galaxies (Giovanelli et al. 1997) or Faber-Jackson (FJ) relation for early type galaxies (Bernardi et al. 2001). These observe galaxy properties in the inner $10h^{-1}$kpc and do not directly measure the virial mass of the halo in which the galaxy sits. At these radii the dynamical effect of baryons on the rotation curves (as well as on the dark matter distribution itself) is important and modifies the relation between the rotation velocity and virial mass. One can however model this assuming stellar mass to light ratio and dark matter profile. The first comes from the stellar population synthesis models and is rather uncertain because of age and IMF (Bruzual & Charlot 1993), while the latter is obtained from the cosmological dark matter simulations and depends on the assumed cosmological model (Bullock et al. 2001). In addition, the response of dark matter to baryon cooling must be included and is often modelled with adiabatic contraction (Blumenthal et al. 1986). Adopting standard values for the dark matter profiles and the stellar mass to light ratios one finds that at L_* the rotation velocity decreases by 1.8 from optical to virial radius both for early and late type galaxies, in good agreement with g-g lensing results (Seljak 2002).

Galaxy-galaxy lensing can also be used to determine the slope β of the relation between mass and luminosity,

$$\frac{M}{M_*} = \left(\frac{L}{L_*}\right)^{\beta}, \tag{5}$$

where M_* is the mass associated with L_* galaxy. Using SDSS data one finds $\beta = 1.4 \pm 0.2$ in red bands, which becomes $\beta^{\rm e} = 1.2 \pm 0.2$ after correcting for a luminosity dependent fraction of early type galaxies in the sample (Guzik & Seljak 2002). This is valid only for early type galaxies between L_* and $7L_*$, which dominate the g-g lensing signal. Note that

this differs significantly from FJ relation, which would predict $\beta \sim 2/3$, but agrees well with the detailed modelling of rotation curves for early type galaxies (Seljak 2002). To determine β around L_* for late type galaxies we cannot use g-g lensing, since the signal is too weak to be detected as a function of luminosity. Instead we use TF relation which gives $L_I \propto v_{\rm opt}^{3.1}$ (Giovanelli et al. 1997). We model the contributions from stellar disk and adiabatic response of dark matter to disk formation to relate between the rotation velocity at the optical radius and virial velocity (or mass). We assume NFW profile with $c_{200} = 10$ and use $\Upsilon_I = 1.5h$, which were shown to reproduce well the virial velocity constraint from g-g lensing at L_* (Seljak 2002). We find $\beta^{\rm l} \sim 1$ around L_*, which is the value we adopt below.

The third parameter that we need is the fraction γ of galaxies that are at the centers of isolated galactic halos, as opposed to larger halos such as groups and clusters. Universal mass functions from N-body simulations only count isolated halos, so one must correct for the fraction of galaxies at a given luminosity that are in larger groups and clusters. This fraction can be determined from the relative contribution to galaxy-galaxy lensing at small and large separations. Above 200-300h^{-1}kpc the signal is dominated by groups and clusters, while below it is dominated by individual galactic halos. Analysis of SDSS galaxy-galaxy lensing data finds $\gamma = 0.72 \pm 0.1$ for early type galaxies and $\gamma = 0.93 \pm 0.1$ for late type galaxies around L_* (Guzik & Seljak 2002). The error includes various systematic uncertanties, of which the galaxy occupation as a function of group and cluster mass is the most important. Thus about 10-30% of L_* galaxies reside in groups and clusters as defined in N-body simulations and one must reduce the galaxy abundance by this fraction when relating it to the halo abundance.

The abundance of galaxies of a given luminosity can be extracted from the luminosity function $dn(L)/d\ln L$, which determines the abundance per logarithmic interval of luminosity. It is often fitted to the Schechter form,

$$\Phi(L) \equiv \frac{dn}{d\ln L} = \phi_* \left(\frac{L}{L_*}\right)^{\alpha+1} \exp(-L/L_*). \tag{6}$$

For L_* galaxies the abundance is $\Phi(L_*) = \phi_*/e$, where $e = 2.718$ is the natural logarithmic base constant. From the early SDSS analysis the values for ϕ_* are 1.46×10^{-2}, 1.28×10^{-2} and 1.27×10^{-2} (with a 10% error) in units of h^3Mpc^{-3} for r', i' and z', respectively, while the values for α are around -1.2 to -1.25 with a few percent error (Blanton et al. 2001). We choose these 3 bands because they show least variation in virial mass to light ratio between early and late type galaxies, minimizing the color dependence of the signal. In addition, the redshift evolution corrections in red bands are smaller than in g' or u'. These have been suggested as one possible reason for the discrepancy between the 2dF luminosity function in b_J and SDSS in g' (Norberg et al. 2001).

The last ingredient that is needed is the fraction of early type ($\xi^{\rm e}$) and late type ($\xi^{\rm l}$) galaxies as a function of magnitude. As shown in Strateva et al. (2001) this fraction depends on luminosity, so that the early type galaxies dominate at the bright end and the late type galaxies dominate at the faint end. This statement is only valid for red bands and a reverse trend is seen in u', while g' shows comparable fractions almost independent of luminosity. In red bands around L_* the fraction is somewhat higher for the early types. While the transition between the early and late types is not well defined the transitional types (S0-Sa) do not dominate the counts, so here we will simply assume a bimodal early/late distribution ($\xi^{\rm e} + \xi^{\rm l}$=1), rather than attempt to model the whole range of stellar ages and morphologies. This could be improved in the future as larger statistical samples are obtained and is particularly important for spirals, which have a larger scatter in the stellar ages. Early type galaxies are more homogeneous and old, so their scatter in stellar mass to light ratio should be small.

We can now combine the above equations to relate halo and galaxy abundances. This gives

$$\frac{dn}{d\ln M} = \frac{dn}{\beta d\ln L} = \frac{\gamma\Phi(L)}{\beta} = \frac{n_{\rm eff}+3}{6}\frac{\bar{\rho}}{M_{200\Omega_m}} f. \tag{7}$$

Using $\bar{\rho} = \Omega_m \rho_{\rm crit}$=$2.77 \times 10^{11} h^2 M_\odot$Mpc^{-3} one finds the minimum mean density at a given luminosity is given by

$$\Omega_m(n_{\rm eff}+3) = 0.3\frac{\xi\gamma}{f\beta}\left(\frac{\Phi(L) M_{200\Omega_m}}{1.4 \times 10^{10} h^2 M_\odot {\rm Mpc}^{-3}}\right). \tag{8}$$

Note that one must add up the contribution from both early and late type galaxies separately, where the two contributions have to be evaluated at equal mass, not luminosity. If there are dark halos without a bright galaxy at the center the above expression become inequality and one can only place a lower limit on $\Omega_m(n_{\rm eff}+3)$. We parametrize this uncertainty with the fraction of dark halos $f_{\rm dh}$, which in general is a function of halo mass.

We can evaluate this expression at several different values for halo mass. The virial mass of an early type galaxy at $L_* = 2.1 \times 10^{10} h^{-2} L_\odot$ in i' is $M_{200\Omega_m} = 1.2 \times 10^{12} h^{-1} M_\odot$ (Guzik & Seljak 2002). Using $\beta^{\rm e} = 1.2$, $\gamma^{\rm e} = 0.72$ and $\xi^{\rm e} = 0.6$ one finds $\Omega^{\rm e}_{\rm m}(n_{\rm eff}+3) = 0.14/(1-f_{\rm dh})$. To this we must add the contribution from late types at the same mass. At L_* their virial mass is $M_{200\Omega_m} = 4.3 \times 10^{11} h^{-1} M_\odot$. Using $\beta^{\rm l} = 1$ one finds that for $M_{200\Omega_m} = 1.2 \times 10^{12} h^{-1} M_\odot$ the corresponding luminosity is $3L_*$. At this luminosity the fraction of late type galaxies in the sample is already small, $\xi^{\rm l} \sim 0.2$. In addition, from the luminosity function in equation (6) one can see the abundance of $3L_*$ galaxies has decreased by a factor of 10 relative to that of L_*. So late type galaxies do not actually add much to the limit above and together we find $\Omega_{\rm m}(n_{\rm eff}+3) = 0.15/(1-f_{\rm dh})$. These constraints are evaluated in i', but one finds similar constraints also in r' and z'. The error budget is dominated by the errors on $M_{200\Omega_m}$, γ and β, which combined give about 30% uncertainty.

We can repeat the same analysis one magnitude above and below L_*. At $L = 2.5L_* = 5.2 \times 10^{10} h^{-2} L_\odot$ the sample is dominated by early type galaxies, so $\xi^{\rm e} \sim 0.8$, $\beta^{\rm e} \sim 1.2$ and $M = 3.6 \times 10^{12} h^{-1} M_\odot$. The fraction of these galaxies in isolated galactic halos is not well determined, but is likely to be larger than at L_* (Guzik & Seljak 2002), so we will assume $\gamma^{\rm e} = 0.9$. This gives $\Omega_{\rm m}(n_{\rm eff}+3) > 0.13/(1-f_{\rm dh})$, where we have ignored the very small contribution from the late type galaxies. The effective slope is about 5-10% higher than at $1.2 \times 10^{12} h^{-1} M_\odot$ (figure 1), so the obtained value is about 20% lower than the value obtained at L_*. If we assumed $\gamma^{\rm e}$ does not differ from that at L_* we find the two estimates are in perfect agreement. This is quite impressive

given that the masses and abundances change by a factor of several. The error is comparable to the error at L_*.

One magnitude below L_* the sample is dominated by late type galaxies, for which we use $\beta^{\rm l} \sim 1$, $M = 1.7 \times 10^{11} h^{-1} M_\odot$ and $\gamma^{\rm l} \sim 0.9$. Adopting $\xi^{\rm l} = 0.8$ leads to $\Omega_m(n_{\rm eff}+3) = 0.11$. To this we have to add the contribution from early types at $M = 1.7 \times 10^{11} h^{-1} M_\odot$. If $\beta^{\rm e} = 1.2$ extends to this mass range this mass corresponds to $L \sim 0.1 L_*$ and at this luminosity the fraction of early type galaxies is about 10%. This increases the above estimate by 20%, so $\Omega_m(n_{\rm eff}+3) = 0.13/(1-f_{\rm dh})$. This estimate is more uncertain, since both $\beta^{\rm l,e}$ and $\gamma^{\rm l,e}$ have not been directly measured over this range. In addition, the large scatter in stellar ages for late type spirals leads to a scatter in mass to luminosity relation. Note that at this mass one expects $n_{\rm eff}+3$ to decrease by about 20% relative to $1.2 \times 10^{12} h^{-1} M_\odot$ (figure 1), so this constraint is actually very similar to the one at L_* based on early type galaxies, even though the masses differ by almost an order of magnitude.

From the above analysis we find that over the range of masses between $1.6 \times 10^{11} h^{-1} M_\odot$ to $3.6 \times 10^{12} h^{-1} M_\odot$ the cosmological constraints on $\Omega_m/(1-f_{\rm dh})$ are all very similar,

$$\Omega_{\rm m}(n_{\rm eff}+3)(1-f_{\rm dh}) = 0.15 \pm 0.05. \quad (9)$$

The good agreement found over a wide range of mass suggests that the shape of the mass function agrees well with the one predicted from cosmological simulations, assuming the fraction of dark halos $f_{\rm dh}$, if different from zero, does not vary over this mass range.

3 DISCUSSION

In this paper we propose a method to derive cosmological constraints from the virial masses and abundance of L_* galaxies and apply it to early SDSS observations. The method differs from other methods using mass to light ratio M/L in that it only uses this information around L_* galaxies and not the overall luminosity density. This sidesteps the uncertainties related to the variation of M/L with luminosity L. The main observational inputs are relation between virial mass and luminosity around L_* as a function of morphological type and the fraction of these galaxies in isolated halos as opposed to groups and clusters, all of which can be extracted from g-g lensing. Another essential ingredient is the luminosity function of galaxies around L_* as a function of morphological type, which can be obtained from the same data as g-g lensing information. The main theoretical input is the halo mass function, which has been shown to be universal by a number of studies (Sheth & Tormen 1999, Jenkins et al. 2001, White 2001). The abundance of halos depends only on the density parameter Ω_m and the effective slope of the linear power spectrum $n_{\rm eff}$ through the combination $\Omega_m(n_{\rm eff}+3)$.

In the absence of dark halos the obtained constraint $\Omega_m(n_{\rm eff}+3) = 0.15 \pm 0.5$ is low compared to the predictions of ΛCDM model with $\Omega_m = 0.3$, $h = 0.7$ and $n = 1$, which gives $\Omega_m(n_{\rm eff}+3) = 0.28$ at these scales. This is excluded by the current constraints, unless a significant fraction of halos is dark over this range of masses. To bring the models into a better agreement with this constraint one can either lower the effective slope or the mean density. The former can be lowered by reducing the primordial spectrum slope n, decreasing the shape parameter Γ (which depends on $\Omega_m = \Omega_{dm} + \Omega_b$, Ω_b and Hubble constant H_0, see e.g. Eisenstein & Hu 1998) or introducing warm dark matter (Bode, Ostriker, & Turok 2001). For example, we find that a model with $\Omega_m = 0.25$, $h = 0.65$, $\Omega_b = 0.04$ and $n = 0.8$ gives a good agreement with the observational constraints, but somewhat lower values of $\Omega_m \sim 0.2$ with $n = 0.9 - 1.0$ are also acceptable. For warm dark matter models, which suppress power on small scales, we compute transfer functions using CMBFAST (Seljak & Zaldarriaga 1996) and find that the effective slope at the galactic mass scale remains almost unchanged for $m_\nu > 500eV$ and drops to $n_{\rm eff} = -2.5$ at $m_\nu = 250eV$. Such low masses are probably excluded from $Ly-\alpha$ forest studies (Narayanan et al. 2000), although this must be confirmed with a more careful error analysis of $Ly-\alpha$ forest constraints.

There are several possible sources of uncertainty that can affect the current constraints derived above. First there is the possibility that the virial masses used here are too low. This is certainly possible for late type galaxies, which have a weak signal in g-g lensing and for which a factor of 2 increase in mass is less than a 2-σ excursion. For early type galaxies the statistical error on the virial mass is 20%, so a factor of 2 excursion is unlikely, although some systematic uncertainties remain in the g-g lensing analysis. The agreement between early and late type galaxies implies that the mass scale must be changed for both types. Furthermore, any increase in the virial mass would affect the agreement between g-g lensing masses and Tully-Fisher or Faber-Jackson relation. A change in mass by a factor of 2 would reduce the optical rotation velocity to virial rotation velocity ratio from 1.8 to 1.4 (Seljak 2002). Within the context of adiabatic compression models such a ratio is likely to be too small to be explained with CDM profiles and stellar mass to light ratios as expected from stellar population synthesis models with reasonable IMF. This possibility would thus require one to give up a successful prediction of CDM models, that of the structure of dark matter halos in the outer parts.

Another possibility is that the relation between the halo mass and luminosity $M \propto L^\beta$ is shallower than assumed here, $\beta \sim 0.6$. This would be the case if virial mass to light ratio was decreasing with luminosity. Both g-g lensing and modelling of optical relations are consistent with $\beta = 1-1.5$ at L_* and above and theoretical models also predict $\beta > 1$ in this regime (e.g. Kauffmann et al. 1999, Benson et al. 2000). While M/L probably decreases with L well below L_*, this is unlikely to be the case over the range of luminosities of interest here. If $\beta^{\rm e} = 1$ instead of 1.2 used here this would increase the lower limit on $\Omega_m(n_{\rm eff}+3)$ by 20%. It seems similarly unlikely that the luminosity function can be wrong by more than 30%, unless there is a large fraction of low surface brightness galaxies that are missed by the SDSS survey. There are still important calibration differences between different surveys, which can cause a mismatch in derived luminosity functions (Norberg et al. 2001), but these are not relevant for our analysis since we derive the g-g lensing relation between light and luminosity using the same sample that is used for luminosity function as well.

Another uncertainty is the division into early and late type galaxies and their associated fractions, which is some-

what artificial, since there is a continuous range of colors and light concentrations in the sample. What is really relevant is the stellar mass of the galaxy, which seems to correlate quite well with the virial mass. For late type galaxies the range of stellar ages is large and this introduces a spread in the luminosity for a given stellar mass, so representing them with a single mass to light ratio may not be accurate. This is less of an issue for early type galaxies, which are very old and for which the spread in the stellar mass to light ratio is small. On the other hand, early type galaxies tend to reside in denser enviroments and the fraction of these in groups and clusters is larger. This correction is also somewhat uncertain, since one must assume how the groups and clusters are populated with these galaxies to determine it (Guzik & Seljak 2002). There is also the issue whether the mass profile of galaxies within groups and clusters differs from that of the same luminosity in the field within inner 100kpc. Observations (Guzik & Seljak 2002) and numerical simulations (Ghigna et al. 2000) suggest they do not differ much, but the uncertainties are still large. The extreme case is when there is actually no mass attached to individual galaxies inside groups and clusters. Then one should use $\gamma^{\rm e} = 1$, which would increase the estimated $\Omega_m(n_{\rm eff} + 3)$ by 30%.

The choice of the virial radius and its associated mass, defined differently by different authors, has a minor effect on our results. From the simulations it is usually defined by friends-of-friends algorithm (assuming a specific value of linking parameter, e.g. 0.2), but it is not always clear how this relates to the specific mean density within the virial radius. For consistency with Jenkins et al. (2001) we define the virial radius as the value where the mean overdensity is $\bar{\delta} = 200$, while the observed masses are usually expressed in terms of overdensity relative to the critical density. For a given halo profile one can convert between the two, but the conversion depends on the assumed density parameter Ω_m. Fortunately the differences are not very important on galactic scales, where halos are highly concentrated and where the mass around the virial radius only slowly grows with radius (logarithmically in the limit of a large concentration where the slope at virial radius approaches -3). For example, the difference in mass between $\bar{\delta} = 200\Omega_m$ for $\Omega_m = 0.4$ and $\Omega_m = 0.2$ in a halo of $c = 12$ is 10%, so this is not the dominating source of error. This effect has been included in the estimates above.

The remaining uncertainty is the fraction of dark halos in the universe. Theoretically one would not expect halos to be dark around L_*, where the efficiency of cooling and star formation is high. This is supported by the fact that for the halos that we do see a large fraction of the baryons within the virial radius has converted into stars (Guzik & Seljak 2002). At lower halo masses, below $10^{11}h^{-1}M_\odot$, the fraction of dark halos may increase because some of the formed disks may be stable against star formation (Verde, Oh, & Jimenez 2002). At higher halo masses above $10^{13}h^{-1}M_\odot$ one enters the group regime, cooling becomes less efficient, and a significant fraction of these halos may not host a bright galaxy at the center. For example, applying the same analysis as in this paper to $7L_*$ early type galaxies one finds the abundance of halos to be several times below that expected from the mass function, a consequence of exponentially decreasing luminosity function at the bright end. This is not surprising, since a large fraction of the groups in this range probably hosts several fainter galaxies rather than a single bright galaxy at the center. It is difficult to determine the abundance of such groups directly from the optical data, since it is not obvious which groups are virialized to satisfy the halo definition in an N-body simulation and which are just a collection of galaxies approaching each other for the first time (as for example the local group). The fact that the constraints derived here agree from $1.6 \times 10^{11}h^{-1}M_\odot$ to $3.6 \times 10^{12}h^{-1}M_\odot$ suggests that over this range of masses the simplest possibility with $f_{\rm dh} = 0$ is consistent with the data.

The prospects to determine the fraction of dark halos directly seem difficult. The only direct way to observe these is through the gravitational lensing, but in the absence of a significant baryon condensation such halos are inefficient strong lenses (Kochanek & White 2001). The halos that do cool and form a disk without making stars could be somewhat more efficient for a given halo mass, but are expected to have lower halo masses. Most of the lenses are bright early type galaxies which both reside in massive halos and have a significant baryonic contribution to the lensing cross-section. It is thus possible that even if some fraction of halos around $10^{12}h^{-1}M_\odot$ exists without a central galaxy they may not have been detected so far with strong lensing. Current surveys such as SDSS may provide better limits on the fraction of dark halos as a function of halo mass.

It is clear from the above discussion that the errors in the current analysis are still quite large, although the fact that the constraints are consistent over a range of masses increases the confidence level of the final result. It is interesting that the constraints obtained are in a good agreement with the cluster gas fraction determination of the matter density (Erdogdu, Ettori, & Lahav 2002) and with the redshift distortions and bias determination from 2dF (Verde et al. 2001). They are also comparable or somewhat higher than those using global M/L ratio (Bahcall et al. 2000, Wilson et al. 2001), although this method, as discussed above, may well be biased. Similarly, a tilted CDM model may help aleviate some of the small scale problems with CDM (Alam, Bullock, & Weinberg 2001). This perhaps indicates the need for a somewhat lower Ω_m or $n_{\rm eff}$ than previously suggested $\Omega_m = 0.3 - 0.4$, $n = 1$ model.

While the error from the method presented here is still large, the prospects to improve it are good. Currently the errors are dominated by observationally determined parameters $M_{200\Omega_m}$, ϕ_*, β and γ. These errors are dominated by statistics and were obtained by using only 5% of the final SDSS sample. With the full sample one can reduce the error on each of these significantly, as well as extend the observable range to the lower luminosity galaxies. With the full sample one can also study the morphological dependence of the mass-luminosity relation in more detail, which as we have shown here plays an important role in the analysis. With spectroscopic information it should be able to extract stellar mass information for each galaxy separately and use g-g lensing to relate the stellar mass to the virial mass directly without splitting the sample into morphological types as done here. Theoretical errors are mainly caused by the accuracy of the mass function over this mass range, but the uncertainty is already at a 10% level and can be further improved with simulations that cover a wider range of cosmological models. The remaining uncertainty is the fraction

of dark halos, which should also be determined with a comparable accuracy (or shown to be negligible). In this case the method presented here may become an accurate test of matter density and slope of the power spectrum on one megaparsec scale.

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